# Effect of a magnetic field on the wake potential in a dusty plasma with streaming ions

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The wake potential of a test dust particulate due to an ion cyclotron wave in a dusty plasma with streaming ions is calculated. The role of the external magnetic field on the periodic attractive forces is clarified. The amplitude of wake potential is reduced because the overshielding by streaming ions is inhibited in the presence of the external magnetic field.

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It is now increasingly clear that the periodic attractive forces among the same polarity dust particulates play an important role in the dust plasma crystal formation. Dust Coulomb crystal formation is attracting a great interest among the plasma physics community in recent years [1–6], with a view to creating new materials for the future. It is not clearly understood why the dust grains of the same charge attract each other and form a crystal lattice in the laboratory conditions.

A number of authors have attempted to explain the periodic attractive forces among the same polarity charges in the literature. Hamaguchi and Farouki [7] considered dielectric polarization of grains in the sheaths around them making them electric dipoles that attract each other. Mohideen *et al.* [8] have explained the attractive force between the columns of grains in the sheath due to the electric-field-induced dipole-dipole interaction giving rise to the lattice structure of dusty-plasma Coulomb crystal formation. Melandso and Goree [9] explained the origin of attraction among dust grains by considering the focusing of ions flowing in the sheath due to deflection of ion orbits by the highly negatively charged dust grains. Another theory [10–13] of wake potential involving the resonant collective interaction of moving grains with the dusty plasma modes, may explain the presence of periodic attractive forces along the line of motion of the dust grains. Vladimirov and Nambu [12] showed that the charged dust particulates in unmagnetized plasmas with finite ion flows can attract each other due to collective interaction involving the ion oscillation in the flow. Low-frequency electrostatic fluctuations [14] as well as pairs of particulates with small separation have also been observed in addition to the formation of the macroscopic Coulomb crystals of solid particles and particle coagulation. A number of studies [10–13] have shown theoretically the particulate attraction to be operative due to the collective interaction of the charged particles with the plasma modes of the dusty plasmas similar to the formation of Cooper pairs [15] in superconductors due to the presence of the low-frequency electrostatic waves.

Recently, Takahashi *et al.* [16] have demonstrated experimentally the formation of dust Coulomb crystals due to wake potential in a plasma with a finite ion flow. The experimental results by optical manipulations agree with the prediction of the importance of the wake potential in dusty plasmas. They showed that a dust particulate located in the up stream of ion flows caused an attractive force on another dust particulate in the lower part. It has been found that many dust particulates are strongly correlated in the vertical direction along the ion flow.

The purpose of this paper is to study the wake potential causing the formation of Coulomb lattice of dust grains in the presence of a static external magnetic field with a continuous flow of ions as in a sheath region of a laboratory dust-crystal experiment. Generally speaking, we have two cases. The first one is that the test particle velocity is much larger than the ion streaming velocity, which was studied by Shukla and Salimullah [17]. The second case occurs when the test dust particulate is stationary. In this paper, we focus on the second case because it corresponds to the real laboratory experiment on plasma crystal. Furthermore, we consider the externally applied uniform magnetic field perpendicular to the shealth plane in Case A, and parallel to the shealth plane in Case B, separately.

## CASE A: $(\hat{z}\|\mathbf{u}_{io}\|\mathbf{B}_o)$

We consider a homogeneous dusty plasma embedded in a uniform external magnetic field perpendicular to the sheath plane where ions are flowing with a constant velocity $u_{io}$ along the magnetic-field lines $(\hat{z}\|\mathbf{u}_{io}\|\mathbf{B}_o)$. We assume that the electrons form a Boltzmann gas at the laboratory temperature, ions are having drift $\mathbf{u}_{io}$, and the highly charged and relatively massive dust grains are taken cold and unmagnetized. Furthermore, we assume the influence of the dust on the dielectric response function can be neglected as we consider the wave interaction in the ion scale. The presence of the charged dust grains only introduces the nonneutrality of the electrons and ion densities in the magnetized dusty plasma. The resonant interaction of the grains and the Doppler-shifted ion-cyclotron wave can give rise to the wake potential under consideration.

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The dielectric response function, in the presence of the low-frequency electrostatic ion-cyclotron wave $(\omega,\mathbf{k})$, is given by

$$\epsilon(\omega,\mathbf{k})=1+\frac{1}{k^2\lambda_D^2}+\frac{k_\perp^2}{k^2}\frac{\omega_{pi}^2}{\omega_{ci}^2-(\omega-k_\parallel u_{io})^2}, \quad (1)$$

where we assume $k_\perp^2 \gg k_\parallel^2$. Throughout this paper, the symbol $\parallel(\perp)$ denotes a quantity parallel (perpendicular) to the ion flow, which is perpendicular to the sheath plane. In Eq. (1), $\lambda_D^2=T_e/4\pi e^2 n_{eo}$, $\omega_{pi}^2=4\pi e^2 n_{io}/m_i$, and $\omega_{ci}=eB_o/m_i c$; $e$, $T_e$, $m_e$, $m_i$, $n_{eo}$, $n_{io}$, and $c$ are the electronic charge, temperature of electrons, mass of an electrons, mass of an ion, number density of electrons, number density of ions, and light velocity, respectively.

Here, we show the limit of applicability of our model. There are a number of scale lengths in the problem such as electron Debye length $(\lambda_D)$, the intergrain distance (d), the electron mean-free-path $(\lambda_{mfp})$, and the characteristic size of the system $(L)$. We assume the following physical domains in the problem $\lambda_D<d\ll L<\lambda_{mfp}$. More explicitly, we are considering a dusty plasma in the presence of an electrostatic ion-cyclotron wave, where $k_\perp v_{te}\ll\omega_{ce}$ (or $\rho_e\ll k_\perp^{-1}$) and $\omega(\cong\omega_{ci})/k_\parallel\ll v_{te}$. Here $\omega_{ce},\rho_e$ are the electron cyclotron frequency and electron Larmor radius, respectively. Under the above two conditions, the thermal electrons can be considered to be unmagnetized, and the electron susceptibility is given by $\chi_e=1/k^2\lambda_D^2$. However, in the perpendicular direction, $\rho_e\ll k_\perp^{-1}$ and $k_\perp\sim\rho_s^{-1}\ll\rho^{-1}$, where $\rho_s=c_s/\omega_{ci}$ and $\rho$ is the perpendicular distance.

The electrostatic potential around a test dust particulate in the presence of the electrostatic mode $(\omega,\mathbf{k})$ in a uniformly magnetized dusty plasma whose response function is given by $\epsilon(\omega,\mathbf{k})$, Eq. (1) is given by [10,18]

$$\Phi(\mathbf{x},t)=\int\frac{q_t}{2\pi^2k^2}\frac{\delta(\omega-\mathbf{k}\cdot\mathbf{v}_t)}{\epsilon(\omega,\mathbf{k})}\exp(i\mathbf{k}\cdot\mathbf{r})d\,\mathbf{k}\,d\omega, \quad (2)$$

where $\mathbf{r}=\mathbf{x}-\mathbf{v}_t t$, $\mathbf{v}_t$ is the velocity vector of the test dust particulate, and $q_t$ is its charge. For the ion-cyclotron wave, the inverse of the dielectric-response function, Eq. (1) is given by

$$\frac{1}{\epsilon(\omega,\mathbf{k})}=\frac{k^2\lambda_D^2}{1+k^2\lambda_D^2}\left[1+\frac{\omega_k^2}{(\omega-k_\parallel u_{io})^2-\omega_{ci}^2-\omega_k^2}\right], \quad (3)$$

where $\omega_k^2=k_\perp^2c_s^2/(1+k^2\lambda_D^2)$, and $c_s=\lambda_D\omega_{pi}$ is the ion sound velocity. Substituting Eq. (3) into Eq. (2) and carrying out the integration following the standard procedures [10–13], we obtain the total potential as the sum of two potentials $\Phi=\Phi_c+\Phi_w$, where the Coulombian part turns out to be $\Phi_c=(q_t/r)\exp(-r/\lambda_D)$.

The non-Coulombian potential involving the collective interaction between the ion-cyclotron wave and a slowly moving or static test dust particulate with $\mathbf{v}_t\|\hat{z}$ can be calculated from

$$\Phi_w(\rho,\xi)=\left(\frac{q_t}{\pi}\right)\int\frac{\delta(\omega-k_\parallel v_t)}{k^2+k_d^2}\frac{\omega_k^2J_o(k_\perp\rho)}{(\omega-k_\parallel u_{io})^2-(\omega_{ci}^2+\omega_k^2)}\times\exp(ik_\parallel\xi)k_\perp\,dk_\perp\,dk_\parallel\,d\omega, \quad (4)$$

where $\xi=z-v_t t$, $k_d=1/\lambda_D$, and $J_o$ is the zero-order Bessel function of the first kind. On carrying out the $\omega$ integrations, we obtain from Eq. (4)

$$\Phi_w(\rho,\xi)=\left(\frac{q_t}{\pi}\right)\int\frac{1}{k^2+k_d^2}\frac{\omega_k^2J_o(k_\perp\rho)}{(k_\parallel u_{io})^2-(\omega_{ci}^2+\omega_k^2)}\times\exp(ik_\parallel z)k_\perp\,dk_\perp\,dk_\parallel. \quad (5)$$

Here, we assume $k_\parallel v_t\approx0$, which corresponds to the second case of stationary test dust particulate mentioned above. Introducing the dimensionless notation $K=k\lambda_D$, we get

$$\Phi_w(\rho,\xi)=\left(\frac{q_t}{\pi\lambda_D}\right)\int\frac{1}{K^2+1}J_o\left(\frac{K_\perp\rho}{\lambda_D}\right)\times\frac{K_\perp^2M^{-2}}{(K_\parallel^2-K_1^2)(K_\parallel^2+K_0^2)}\exp\left(\frac{iK_\parallel z}{\lambda_D}\right)K_\perp dK_\perp dK_\parallel, \quad (6)$$

where

$$K_{0,1}^2=\pm\left(1-\frac{M^{-2}}{f}+K_\perp^2\right)\Big/2+\left[K_\perp^2M^{-2}+\frac{M^{-2}(1+K_\perp^2)}{f}+\left(1-\frac{M^{-2}}{f}+K_\perp^2\right)^2\Big/4\right]^{1/2}. \quad (7)$$

Here, $M=u_{io}/c_s$ is the Mach number, and $f=\omega_{pi}^2/\omega_{ci}^2=c^2/v_A^2\gg1$, $c$ and $v_A$ are light velocity and the Alfven velocity, respectively.

For $1>K_\perp^2$, $f=c^2/v_A^2\gg1$, and $M>1$, the dominant contribution of Eq. (7) reduces to

$$K_0^2\approx1, \quad K_1^2\approx\frac{K_\perp^2+\dfrac{1}{f}}{M^2}. \quad (8)$$

We must note that the characteristic wave number of the ion cyclotron wave $k_\perp\approx\rho_s^{-1}$, where $\rho_s=c_s/\omega_{ci}$. Accordingly, we find $K_\perp^2\approx1/f$. We now obtain the oscillating wake potential that originates from the residues at the poles at $K_\parallel=\pm K_1$ in Eq. (6). We note that contribution from the poles at $K_\parallel=\pm iK_0$ shows the change of the effective Debye length in plasmas. Then we obtain

$$\Phi_w(\rho=0,z)=\frac{-2q_t}{M\lambda_D}\int_0^1\frac{K_\perp^3}{(K_\perp^2+1/f)^{1/2}}\sin\left\{\frac{(K_\perp^2+1/f)^{1/2}z}{M\lambda_D}\right\}dK_\perp. \quad (9)$$

By repeating partial integrations, we get from Eq. (9)

$$\Phi_w(\rho=0,z)=\frac{2q_t}{z}\left[\cos\left\{\frac{(1+1/f)^{1/2}z}{M\lambda_D}\right\}+\frac{2M\lambda_D}{z}\left(\sqrt{\frac{1}{f}}\sin\left\{\frac{z}{f^{1/2}M\lambda_D}\right\}-\sqrt{1+\frac{1}{f}}\sin\left\{\frac{(1+1/f)^{1/2}z}{M\lambda_D}\right\}\right)+\frac{2M^2\lambda_D^2}{z^2}\left(\cos\left\{\frac{z}{f^{1/2}M\lambda_D}\right\}-\cos\left\{\frac{(1+1/f)^{1/2}z}{M\lambda_D}\right\}\right)\right]. \tag{10}$$

For $z>M\lambda_D$, the dominant wake potential comes from the first term of Eq. (10) as

$$\Phi_w(\rho=0,z)\approx\frac{2q_t}{z}\cos\left(\frac{z}{L_0}\right), \tag{11}$$

where $L_0=M\lambda_D(1+1/f)^{-1/2}$ is the effective length. Equation (11) shows the wake potential due to ion-cyclotron waves in a dusty plasma when the test particulate is stationary. Here, we compare Eq. (11) with that of unmagnetized plasma case [Eq. (11) of Ref. [12]], where the dominant wake potential is given by $\Phi_w(\rho=0,z)\approx 2q_t/z(1-M^{-2})\cos(z/L_s)$, here $L_s=\lambda_D\sqrt{M^2-1}$. The amplitude ratio of wake potential [$R_0$=(amplitude for magnetized plasma for case A)/(amplitude for unmagnetized plasma)] reduces to

$$R_0=(1-M^{-2})^{3/2}<1. \tag{12}$$

Furthermore, the ratio of effective length is given by $L_0/L_s=(1+1/f)^{-1/2}(M/\sqrt{M^2-1})$, which is larger than unity, because $f=\omega_{pi}^2/\omega_{ci}^2=c^2/v_A^2\gg 1$. Thus, the external magnetic field parallel to the ion flow direction reduces the amplitude of wake potential when $M>1$.

### CASE B: ($\hat{z}\|\mathbf{u}_{io}\perp\mathbf{B}_o$)

Next we consider the case where the external magnetic field is in the $x$ direction with $\mathbf{k}=k_x\hat{x}+k_y\hat{y}+k_\|\hat{z}$, where $\mathbf{k}_\|=k_z\hat{z}\|\mathbf{u}_{io}$ and $k_\perp=\sqrt{k_x^2+k_y^2}$. Here, the symbol $\|(\perp)$ denotes a quantity parallel (perpendicular) to the ion flow, which is perpendicular to the external magnetic field.

In obtaining the ion dielectric response, the approximation $k_\|\gg k_y$ is made. This means that dominant wave vector perpendicular to the external magnetic field associated with the ion cyclotron wave is in the z direction parallel to the ion-beam direction. This is often used in the modified two-stream instability (cross field instability) [19]. This approximation is also related to the validity of neglect of $\mathbf{E}\times\mathbf{B}$ drift. We have here chosen the dc magnetic field perpendicular to the ion streaming direction. The electric field ($E$) is in $z$ direction and $B_o$ is in $x$ direction. Thus, $E*B$ drift is in $y$ direction and drift velocity is given by $v_d=cE/B_o$. Here, we assume $k_\|/k_y\gg v_d/u_{io}$. For example, for considerably large $B_o$, $v_d$ is small compared to $u_{io}\sim c_s$, and may be neglected. For example, in Ref. [20], various parameters are $B_o=4$ T, the radial electric field $E_r=100$ V/m, and the ion streaming velocity $u_{io}=10^3$ m/s. Thus, we find $v_d=25$ m/s $\ll u_{io}=10^3$ m/s. In other words, we focus our attention to study the vertical structure of the wake potential associated with the ion stream. However, under the reverse condition $k_\|/k_y\ll v_d/u_{io}$, the $E*B$ drift would give an important effect on the structure in the horizontal plane. For example, several laboratory experiments [21–23] have shown the effects of poloidal ion flows associated with $E*B$ drift on dust particle behavior in magnetized plasmas. They explained the rotation of the dust particle in terms of small radial $E$ component and the vertical magnetic field $B_o$. The quantitative estimate of this rotation is yet to be estimated and is beyond the scope of the present paper.

Then, under the above appoximation, the dielectric response function, in the presence of the low-frequency electrostatic ion-cyclotron wave ($\omega,\mathbf{k}$), is given by

$$\epsilon=1+\frac{1}{k^2\lambda_D^2}+\frac{k_\|^2}{k^2}\frac{\omega_{pi}^2}{\omega_{ci}^2-(\omega-k_\|u_{io})^2}. \tag{13}$$

After the similar analysis as in case A, we obtain the non-Coulombian potential as

$$\Phi_w(\rho,z)=\left(\frac{q_t}{\pi\lambda_D}\right)\int\frac{1}{K^2+1}J_o\left(\frac{K_\perp\rho}{\lambda_D}\right)\times\frac{K_\|^2M^{-2}}{(K_\|^2-K_1^2)(K_\|^2+K_0^2)}\exp\left(\frac{iK_\|z}{\lambda_D}\right)K_\perp dK_\perp dK_\|, \tag{14}$$

where

$$K_{0,1}^2=\pm\left(1-M^{-2}+K_\perp^2-\frac{M^{-2}}{f}\right)\Big/2+\left[\frac{M^{-2}}{f}(1+K_\perp^2)+\left(1-M^{-2}+K_\perp^2-\frac{M^{-2}}{f}\right)^2\Big/4\right]^{1/2}. \tag{15}$$

For $M>1$, $M^{-2}/f=\omega_{ci}^2\lambda_D^2/u_{io}^2\ll 1$, and $K_\perp^2<1$, we find the dominant contribution of Eq. (15) as

$$K_0^2\approx 1,\quad K_1^2\approx\frac{1}{f(M^2-1)}. \tag{16}$$

Furthermore, we get the wake potential from the contribution at the poles at $K_\|=\pm K_1$ in Eq. (14)

$$\Phi_w(\rho=0,z)=\frac{-q_t}{M\sqrt{M^2-1}}\left(\frac{\omega_{ci}}{u_{io}}\right)\sin(z/L_1), \tag{17}$$

where $L_1 = \sqrt{f(M^2-1)}\lambda_D$ is the effective length. When the externally applied uniform magnetic field is parallel to the shealth plane, the wake potential is a sensitive function of $M$ and $f$. The amplitude ratio of wake potential $[R_1 = (\text{amplitude for magnetized plasma for case B at } z = M\lambda_D)/(\text{amplitude for unmagnetized plasma})]$ is

$$R_1 = \frac{\sqrt{M^2-1}}{2M^3\sqrt{f}} \ll 1. \tag{18}$$

Furthermore, the ratio of effective length is given by $L_1/L_s = \sqrt{f}$), which is much larger than unity, because $f = \omega_{pi}^2/\omega_{ci}^2 = c^2/v_A^2 \gg 1$.

This means that the wake potential with ion-cyclotron waves becomes weak and the effective length for magnetized plasma is large under the laboratory plasma condition because $f \gg 1$. The reason why the wake potential becomes weak for magnetized plasma can be explained as follows. The mechanism of attractive forces due to wake potential is quite similar to that of the Cooper pairing. The physical mechanism is the overshielding caused by streaming ions for downstream direction from the test static negatively charged dust particulate. For the magnetized plasmas considered here, the ions motion is more or less influenced by the external magnetic field. Thus, the condition for the overshielding due to ions, which is necessary for the appearance of the attractive forces, is not easily satisfied.

To conclude, we have studied the effect of magnetic field due to ion-cyclotron waves on the wake potential ($\Phi_w$) in a dusty plasma with streaming ions. We focused our attention on the situation when the test dust particulate is stationary, because it corresponds to the real laboratory experiment on plasma crystal. First, we considered the externally applied uniform magnetic field perpendicular to the shealth plane in case A. As is shown by Eq. (12), the amplitude of the wake potential for a magnetized plasma is small as compared with that of an unmagnetized plasma. Furthermore, the effective length for the magnetized case becomes larger. Next, we studied the externally applied uniform magnetic field parallel to the shealth plane in case B. The attractive wake potential given by Eq. (17) is much smaller than that for unmagnetized case. It is found the effective length becomes much larger for magnetized plasmas. Thus, depending upon the strength of the magnetic field, a given dust crystal structure may vanish because of the application of the transverse magnetic field, resulting into a new method of phase transition in the dust-Coulomb crystals.

The total potential is given by the sum of two potentials, $\Phi = \Phi_c + \Phi_w$, where the Coulombian part turns out to be $\Phi_c = (q_t/r)\exp(-r/\lambda_D)$, which is the same for unmagnetized plasmas [12]. As is found by this paper, the attractive wake potential ($\Phi_w$) is small and the effective length becomes large for magnetized plasmas when the external uniform magnetic field is applied. Thus, we may conclude from our paper that the spacing between dust particulates in the vertical direction along the ion flow in dust-Coulomb crystal becomes large for magnetized plasmas. These predictions should be compared with the future laboratory dust-crystal experiment.

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[1] Y. Hayashi and K. Tachibana, Jpn. J. Appl. Phys., Part 2 **33**, L804 (1994).
[2] H. Thomas, G. E. Morfill, V. Demmel, J. Goree, B. Feuerbacher, and D. Möhlmann, Phys. Rev. Lett. **73**, 652 (1994).
[3] J. H. Chu and I. Lin, Phys. Rev. Lett. **72**, 4009 (1994).
[4] A. Melzer, T. Trottenberg, and A. Piel, Phys. Lett. A **191**, 301 (1994).
[5] H. Totsuji, T. Kishimoto, and C. Totsuji, Phys. Rev. Lett. **78**, 3113 (1997).
[6] N. Sato, G. Uchida, R. Ozaki, and S. Iizuka, in *Physics of Dusty Plasmas*, edited by M. Horanyi, S. Robertson, and B. Watch (AIP, New York, 1998), p. 239.
[7] S. Hamaguchi and R. T. Farouki, Phys. Rev. E **49**, 4430 (1994).
[8] U. Mohideen, H. U. Rahman, M. A. Smith, M. Rosenberg, and D. A. Mendis, Phys. Rev. Lett. **81**, 349 (1998).
[9] F. Melandso and J. Goree, J. Vac. Sci. Technol. A **14**, 511 (1996).
[10] M. Nambu and H. Akama, Phys. Fluids **28**, 2300 (1985).
[11] M. Nambu, S. V. Vladimirov, and P. K. Shukla, Phys. Lett. A **203**, 40 (1995).
[12] S. V. Vladimirov and M. Nambu, Phys. Rev. E **52**, R2172 (1995).
[13] O. Ishihara and S. V. Vladimirov, Phys. Plasmas **4**, 69 (1997).
[14] J. H. Chu, J. B. Du, and I. Lin, J. Phys. D **27**, 296 (1994).
[15] P. G. de Gennes, *Superconductivity of Metals and Alloys* (Benjamin, New York, 1966), p. 102.
[16] K. Takahashi, T. Oishi, K. Shimomai, Y. Hayashi, and S. Nishino, Phys. Rev. E **58**, 7805 (1998).
[17] P. K. Shukla and M. Salimullah, Phys. Plasmas **3**, 3858 (1996).
[18] N. A. Krall and A. W. Trivelpiece, *Principles of Plasma Physics* (MacGraw-Hill, New York, 1973), p. 562.
[19] R. C. Davidson, *Basic Plasma Physics I*, edited by M. N. Rosenbluth and R. Z. Sagdeev (North-Holland, New York, 1983), p. 583.
[20] N. Sato, G. Uchida, T. Kaneko, S. Shimizu, and S. Iizuka, Phys. Plasmas. (to be published).
[21] H. Fujiyama, H. Kawasaki, S. C. Yang, and Y. Matsuda, Jpn. J. Appl. Phys., Part 1 **33**, 4216 (1994).
[22] S. Nonomura, N. Ohno, and S. Takahashi, Jpn. J. Appl. Phys., Part 1 **36**, 877 (1997).
[23] U. Konopka, D. Samsonov, A. V. Ivlev, J. Goree, V. Steinberg, and G. E. Morfill, Phys. Rev. E **61**, 1890 (2000).